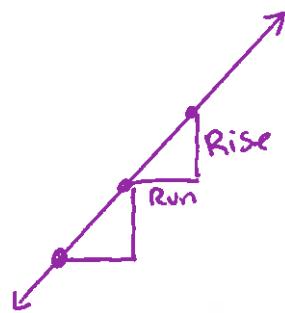


"Steepness"

Equations of Lines

Slope of a line through the points (x_1, y_1) and (x_2, y_2) :

$$m = \frac{\text{Rise}}{\text{Run}} = \frac{y_2 - y_1}{x_2 - x_1}$$



Example: Find the slope of the line through the points $(4, 8)$ and $(-2, -1)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 8}{-2 - 4} = \frac{-9}{-6} = \boxed{\frac{3}{2}}$$

Equation of a Line (2 Forms)

Slope-Intercept Form

$$y = mx + b$$

Point-Slope Form

$$y - y_1 = m(x - x_1)$$

Example: Find an equation for the line passing through the point $(1, -2)$ with a slope of $3/2$.

$$\begin{aligned} m &= \frac{3}{2} \\ \text{Point} &= (1, -2) \end{aligned}$$

Point-Slope $\rightarrow y - (-2) = \frac{3}{2}(x - 1)$

$y + 2 = \frac{3}{2}x - \frac{3}{2}$

$y = \frac{3}{2}x - \frac{7}{2}$ Slope-intercept

Example: Find the equation of the line passing through the points $(4, 8)$ and $(-2, -1)$.

$$m = \frac{-1 - 8}{-2 - 4} = \frac{-9}{-6} = \frac{3}{2}$$

Pick either pt \rightarrow Point-Slope $\rightarrow y - 8 = \frac{3}{2}(x - 4)$ or $y - 8 = \frac{3}{2}x - 6$

Slope intercept $\rightarrow y = \frac{3}{2}x + 2$

$$\begin{aligned} y + 1 &= \frac{3}{2}(x + 2) \\ y + 1 &= \frac{3}{2}x + 3 \\ y &= \frac{3}{2}x + 2 \end{aligned}$$

Parallel and Perpendicular Lines

(Parallel) Theorem: parallel lines have = slopes.

(Perpendicular) Theorem: perpendicular lines have negative reciprocal slopes.

Example: Find the equations of a line parallel and a line perpendicular to the line $-3x + 2y = 4$, each passing through the point $(3, 0)$.

First: find the slope:

$$\begin{aligned} -3x + 2y &= 4 \\ 2y &= 3x + 4 \\ y &= \frac{3}{2}x + 2 \end{aligned}$$

Slope

Parallel Line:

$$m = \frac{3}{2}$$

$$\text{Point} = (3, 0)$$

$$\begin{aligned} y - 0 &= \frac{3}{2}(x - 3) && \leftarrow \text{pt-slope} \\ &\quad \text{or} \\ y &= \frac{3}{2}x - \frac{9}{2} && \leftarrow \text{slope intercept} \end{aligned}$$

Perpendicular Line:

$$m = -\frac{2}{3} \leftarrow \text{neg reciprocal}$$

$$\text{Point} = (3, 0)$$

$$\begin{aligned} \text{pt slope} \rightarrow y - 0 &= -\frac{2}{3}(x - 3) \\ \text{or} \\ \text{slope intercept} \rightarrow y &= -\frac{2}{3}x + 2 \end{aligned}$$

Vertical & Horizontal Lines

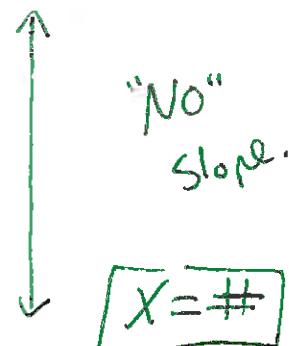
Horizontal



"Zero" slope.

$$y = \#$$

Vertical



Example: Write the equation of the line through the points $(2, 3)$ and $(5, 3)$.

$$m = \frac{3-3}{5-2} = \frac{0}{3} = 0 \leftarrow \begin{matrix} \text{horiz.} \\ \text{line.} \end{matrix}$$

$$y = 3$$

Example: Write the equation of the line through the points $(-3, 4)$ and $(-3, 1)$.

$$m = \frac{1-4}{-3 - (-3)} = \frac{-3}{0} = \text{undefined}$$

Vert, line

$$x = -3$$